Modelling the Impact of Taxes and Wealth Increments on Labour Supply

Modelización del Impacto de los Impuestos y los Incrementos de Capital en la Oferta Laboral

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Abstract
This paper presents a comprehensive work-leisure model designed to examine the impact of wealth changes on work supply. By incorporating both monetary and non-monetary costs, as well as financial and non-financial benefits, the model conducts a thorough cost-benefit analysis, enabling individuals to choose activities that yield the highest remuneration. The theoretical analysis reveals that taxes have a relatively minor effect on individuals within lower income brackets or levels, but their impact grows exponentially as agents accumulate more capital. Consequently, the loss of efficiency resulting from taxes is significantly greater in higher income brackets compared to lower ones. Moreover, considering the diminishing marginal utility of monetary units, a higher standard of living, *ceteris paribus*, tends to correspond to a reduction in the number of hours worked. Overall, this research provides valuable insights into the relationship between wealth changes, taxation, and workers' labour supply decisions, contributing to a deeper understanding of economic behaviour and decision-making.

Keywords: cost-benefit analysis, productivity, hours worked, efficiency loss, taxes.

JEL Classifications: D7, H21, J21

Resumen
En este estudio, proponemos un modelo integral de trabajo-ocio con el objetivo de analizar el impacto de los cambios en la riqueza en la oferta laboral. Incluimos en el análisis costes monetarios y no monetarios, así como los beneficios financieros y no financieros en el modelo, llevando a cabo un análisis coste-beneficio en el cual los actores optan por las actividades que les remuneran más. El análisis teórico concluye que los impuestos tienen un efecto pequeño en los primeros tramos o niveles de ingresos, pero crece de manera exponencial a medida que el agente adquiere más capital, con la consiguiente pérdida de eficiencia siendo mucho mayor en los tramos más altos que en los más bajos. De manera similar, la curva se caracteriza por describir una utilidad marginal decreciente de las unidades monetarias, por lo que, manteniendo las demás variables constantes, un mayor nivel de vida tiende a llevar a un menor número de horas trabajadas. En conjunto, esta investigación proporciona conocimientos valiosos sobre la relación entre los cambios en la riqueza, la tributación y las decisiones de oferta laboral de los trabajadores, contribuyendo a una comprensión más profunda del comportamiento económico y la toma de decisiones.

Palabras clave: análisis coste-beneficio, productividad, horas laborales, pérdida de eficiencia, impuestos.

Clasificaciones JEL: D7, H21, J21
1. Introduction

Numerous studies have been carried out on the appropriateness of reducing the number of hours worked by workers and the policies that could be pursued to improve the welfare of society. These address the environmental consequences (Jackson, 2009; Coote, Franklin, Simms, 2010; Victor, 2010), the stimulative effect of shorter hours on recovery from a crisis (Alesina, Glaeser, Sacerdote, 2005; Messenger, Lee, McCann, 2007; Taylor, 2011), etc. In this article, we focus on the most important and general issues that may be of interest to policymakers. Mainly, we base our analysis on a detailed model in Section 2 and conclude the virtues and disadvantages in efficiency and number of hours worked produced by different taxes (progressive, regressive, leave...), rising living standards and individuals’ time preference (their inclinations concerning time “repression”). The results of our research point to the existence of an inverse relationship between hours worked and tax burden, the standard of living and capacity for personal effort and savings (high time preference).

The economic literature today bases its models of work and leisure on a dichotomy that functions as an axis: agents are only interested in the remuneration they obtain from their work, with their leisure activities being the only ones that provide profits (Moffitt, 2002; Grogger and Karoly, 2005). Other authors even argue that work is always a source of disutility (Spencer, 2003; Spencer, 2009). With respect to neoclassical models, they also fail to take into account possible utilities from productive activities as well as disutilities from leisure activities (Schumacher, 1973; Scitovsky, 1976; Applebaum, 1992; Frey, 1997; Thomas, 1999; Frey and Stutzer, 2002; Layard, 2006; Frey, 2008; Spencer, 2009; Russo, 2012). However, there is also a growing development of models and avenues of research that consider how a worker can enjoy their work —apart from their financial remuneration— and thus that their leisure is not the sole provider of utilities (Lane, 1992; Spencer, 2014; Kaplan, Schulhofer-Wohl, 2018). This study seeks to contribute to such a position.

This article consists of a total of four sections, the first being this introduction. In Section 2, we present the model. In Section 3, we analyse the theoretical consequences accruing from the model in the aspects that are potentially of most interest to society and policymakers. In Section 4, we conclude with the summary and results to be drawn from the study.
2. The model

Work is no different from any other economic activity, in the sense that all activities have both monetary and non-monetary utilities and disutilities. Thus, for the subsequent analysis, we will consider that every actor (an economic agent that acts according to praxeology principles, maximizing utility) will have a utility function $\mu$ for an activity $\alpha$ in an instant $\tau$ of time such that:

$$\mu(\alpha, \tau) = i(\alpha, \tau) + \delta(\alpha, \tau) - \phi(\alpha, \tau) - \varepsilon(\alpha, \tau)$$

(1)

We also consider an effective utility function $\psi$ which considers the above utility function if and only if the activity can be performed (determined by a binary function $\vartheta$):

$$\psi(\alpha, \tau) = \vartheta(\alpha, \tau) \times \mu(\alpha, \tau)$$

(2)

Where:

- $\alpha \equiv$ endogenous variable. It represents an activity according to its index within the set of possible activities imaginable by the actor. For example, if activity 1 is “washing the dishes”, then if $\alpha = 1$, $\alpha$ shall refer to the activity of “washing the dishes”.

- $\tau \equiv$ exogenous variable. It is time, measured in a given time unit. Depending on which magnitude is used (seconds, hours, days...) a different time horizon will be addressed. If a subscript is not specified, i.e. it is an integer (representing a given time or value of $\tau$, such that smaller indices symbolise instants prior to larger indices), it will be a generic magnitude of time. That is to say, the mathematical expression addressed will be true for any unit chosen (seconds, hours, days...).

- $\mu \equiv$ function. It measures the marginal utility in terms of monetary units of an activity $\alpha$ when it has been running for $\tau$ time units. When no money is involved in the realization of some task, the marginal utility becomes the maximum amount that the actor would be willing (1) to lose in order to perform the activity, and (2) to pay to not undergo the parts he deems as uneasy or undesirable.

- $i \equiv$ function. It returns the marginal utility that the actor assigns to the number of remaining monetary units to be received, either secured or speculatively, through the successful completion of the activity $\alpha$.

- $\delta \equiv$ function. Returns the amount the agent is willing to pay to perform the activity $\alpha$ at the moment $\tau$. In conjunction with $i$, is the total value assigned by the agent to carry out the activity $\alpha$ in monetary terms.
- $\phi \equiv$ function. Calculate the opportunity cost of performing $\alpha$ (expressed in monetary units) for the corresponding actor at a given point in time $\tau$.

- $\epsilon \equiv$ function. Calculate the effort of the corresponding actor when performing the activity $\alpha$ in an instant $\tau$, represented by how many monetary units you would be willing to pay to disengage from the activity or part of it.

- $\psi \equiv$ function. It measures effective marginal utility only when the activity in question can be carried out (i.e. it has a value of $\psi(\alpha, \tau) \neq 0$ for any activity that may be carried out and $\psi(\alpha, \tau) = 0$ provided that the activity in question cannot be perpetrated at that particular time).

- $\vartheta \equiv$ binary function. It returns 1 or 0, in case its input is an activity that can or cannot be performed at time $\tau$, respectively. In short, it is a function that considers 0 any activity that is not within the actor’s reach to perform, such as (for most people) a trip to the moon, teleportation to his office or going to a concert outside the time the band is playing. In other words, $\vartheta(\alpha) = 1$ if $\alpha$ is realizable and $\vartheta(\alpha) = 0$ in any other case.

Under the model, an actor $\rho$ will only carry out an activity for a certain period of time as long as $\omega(\alpha, \tau) + \delta(\alpha, \tau) - \phi(\alpha, \tau) - \epsilon(\alpha, \tau) > 0$. This is easily explained given a hierarchy of purposes ordered from most to least important. If a person has to decide for carrying out the activity $\alpha_1$ or $\alpha_2$, the person will analyse the advantages and disadvantages of both (assuming constraints in the actor’s action and some capability of exerting free will). He or she will see what he or she receives or expects to receive in monetary terms — be it through a job or by starting a business, looking for a possible profit — (\omega(\alpha, \tau)) and what he obtains in spiritual, psychological, physiological and, in general, what he receives of value that is not strictly monetary in nature. Sherman and Shavit (2013) have studied ways of measuring these factors as a sort of “immaterial sustenance”, mainly through surveys.

These positive benefits or externalities concomitant to the corresponding activity are represented as what the actor would be willing to pay to receive such non-monetary rewards (or goods) (\delta(\alpha, \tau)). In the same way, it will proceed to assess what it would have to give up at that instant in time ($\tau$) to undertake the desired action (\phi(\alpha, \tau)); i.e. the opportunity cost at that time of carrying out the desired task. Finally, he/she will also weigh up how much effort is involved in carrying out the activity. (\epsilon(\alpha, \tau)). The latter factor is more difficult to quantify in monetary terms, as the wear and tear tend to be mostly cognitive and/or physical. However, it is defined here as the amount that one would be willing to pay to give up certain parts or even the whole activity. A very illustrative example is that of a tourist: he loves the trip he is taking, but does not like to carry so much luggage, which he perceives as a negative part (a cost in effort or, if delegated, financial) of the activity. He would not give up visiting abroad because of this, although he would not mind having a servant carry his things from one place to another. The amount he would be willing to pay for the services of this hypothetical servant we consider as the perceived disutilities of the servant. In other words, the financial and effort cost of the activity.
The case where an individual would have $\mu(\alpha, \tau) < 0$ for every $\alpha$ with a given $\tau$ is not considered here. Or, more precisely, when $\psi(\alpha, \tau) \leq 0$ for every $\alpha$ with $\tau \in \mathcal{H}$, where $\mathcal{H}$ is an interval representing the individual’s time horizon or preference (discussed later). If so, it is considered that the agent would choose to end his life, not considering any of the activities he has at his disposal or is considering carrying out as rewarding or beneficial in any conceivable aspect. This is in line with the reflection carried out by Arthur Schopenhauer ([1819] 2011, p. 509), who affirmed that:

Suicide, the actual doing away with the individual manifestation of will, differs most widely from the denial of the will to live, which is the single outstanding act of free-will in the manifestation, and is therefore, as Asmus calls it, the transcendental change. This last has been fully considered in the course of our work. Far from being denial of the will, suicide is a phenomenon of strong assertion of will; for the essence of negation lies in this, that the joys of life are shunned, not its sorrows. The suicide wills life, and is only dissatisfied with the conditions under which it has presented itself to him. He therefore by no means surrenders the will to live, but only life, in that he destroys the individual manifestation. He wills life—wills the unrestricted existence and assertion of the body; but the complication of circumstances does not allow this, and there results for him great suffering [emphasis added].

The actor’s scrutiny is subjective and can vary from moment to moment. The example is especially clear when comparing childhood with adulthood when $\tau_1 \ll \tau_2$. The person $\rho$ as a child ($\tau_1$) might assign a negligible value to the activity whereas as an adult ($\tau_2$) he or she might devote his or her life to it. See the adolescent who wants to achieve a PhD in Physics, but ends up training and working as a football referee. These variations in the subjective valuation of the same activity depending on when it is perpetuated result in the dynamic nature of the variables involved in the valuation of the usefulness of an activity. Thus, the income from a job and the total monetary profits accrued ($\kappa(\alpha, \tau)$) may vary whether the hours worked are ordinary (those established in the contract) or extraordinary (outside the working day); the benefits or externalities ($\delta(\alpha, \tau)$) are perceived differently depending on the actor’s preferences at the time; the opportunity cost ($\phi(\alpha, \tau)$) of a father to play with his children will diverge between his working hours and his leisure hours; and the effort ($\varepsilon(\alpha, \tau)$) experienced by an adult is different from that experienced by an elderly person.

All this determines whether or not an agent decides to perform an action, but it does not specify how many units of time he/she will spend on one activity $\alpha_1$ and another $\alpha_2$. If $\alpha_1$ is activity 1, which symbolises work; and $\alpha_2$ is activity 2, which represents leisure, the fact that the actor values more the first marginal units of income received from his salary and these exceed the marginal profits earned from spending time with his children does not make him spend the whole day in his office. Thus, we have to develop an analysis scheme that allows us to identify the particular decisions that the actor makes at each moment and that determine the bulk of the actions that he ends up taking as a whole. However, in order to do so, we must first take into account the forms traced by the four essential functions of which the utility function $\mu$ is composed.
2.1. Internal functions

First, we define monetary units as having an exponential diminishing marginal utility (see Figure 1), as to obtain the same relative benefit from an increase in capital it must be much higher than the previous one. Consider the relative change of a final state $m$ with respect to an initial $m_0$:

$$\zeta(m, m_0) = \frac{m - m_0}{m_0}$$  \hspace{1cm} (3)

If we assign $m_0 = 1$ and $m = 2$, then $\zeta(m, m_0) = 1$. In order to maintain $\zeta(m, m_0)$ fixed at the constant value 1, and taking $m_0$ as the last $m$, then we have that the upcoming values of $m$ grow exponentially with the order $O(2^n)$. This goes within the lines of the law of diminishing marginal utility (Sevilla, 2020). In sum, the agent values each additional pound sterling, shilling or penny much less the higher his income in a given interval $\mathcal{H}$, and the character of this diminishing effect is very accelerated, so that (1) the slope will be negative, thus having $\zeta'(\alpha, \tau) \leq 0$; and (2) the curve formed will be concave downwards and thus $\zeta_1'(\alpha, \tau) > \zeta_2'(\alpha, \tau)$.

Second, $\delta(\alpha, \tau)$ is usually a low volatility value. For example, if a driver passes through a spot that is unusually attractive to the eye, drawing $\delta(\alpha, \tau)$ in a graph we could observe that at that instant in time the function makes a small upturn, and then returns to its mean after passing the beautiful landscape. In practice, it is the mean and trend of $\delta(\alpha, \tau)$ that is relevant, and to a large extent could be considered constant throughout the pursuit of a given activity (Csikszentmihalyi, 2008) barring significant variations in personal preferences from one moment to the next (e.g. a politician leaving a party after learning of a serious and sudden case of corruption).
Marginal utility of monetary units over time. Each additional income is perceived as less valuable by the actor.

Third, we define $\phi(\alpha, \tau)$ as showing a behaviour identical to that of $\delta(\alpha, \tau)$ and even less volatile. Only under substantial changes in the information available could the opportunity cost of performing the corresponding activity change. This also makes intuitive sense, as the perceived value of options will not be modified if the actor’s preferences or options themselves do not vary greatly in short periods of time, as we further consider being the case. For example, it would be altered when a clerk gets a promotion or a street sweeper learns that he has won the lottery, the opportunity cost of doing the corresponding activity might change.

Fourth, we can consider that each additional unit of time spent carrying out an activity $\alpha$, other things being equal and except for very specific tasks, declines at the beginning and increases uninterruptedly $\varepsilon(\alpha, \tau)$ from its minimum. In short, the shape that $\varepsilon(\alpha, \tau)$ draws when represented over time is parabolic with a concave upward curvature. This is because psychological friction is always encountered when breaking with inertia and switching from one activity to another (Lepine, Colquitt, Erez, 2000; Cepeda, Kramer, Gonzalez de Sather, 2001), whereas it is easier to stay with the same activity as long as the challenge is aligned with personal capacity (Csikszentmihalyi, 2008). For example, as we perform the initial exercises in our sport, our body becomes more and more prepared for the action and intensity that follow shortly after.

2.2. Time allocation

In this model we assume that actors are rational, meaning that they wish to maximise the utilities they receive from each activity on a given day. In short, every agent $\rho$ at a time $\tau$ is faced with the problem of choosing to which activity to devote what amount of time units (seconds, minutes, hours...):

$$\max_{\alpha} \psi(\alpha, \tau)$$

(4)

Faced with such a dichotomy, the actor will take stock (consciously or unconsciously) of what he sees and expects to generate greater utility throughout his day. Thus, an agent $\rho$, regardless of the characteristics of his time horizon (whether it is myopic, moderately broad or considerably broad; that is, whether he uses $\tau_s$, $\tau_d$ or $\tau_y$, respectively, as a measure), will underpin his daily tasks one by one by performing short-term analyses to decide which activity to devote his efforts to. This is important because the aggregate of these decisions will add up to the total amount that the agent will devote to each of the possible activities ($\mathbb{A}$). In sum, at the “micro” level (instant by instant) each actor will decide which activity it
is convenient for him to carry out now, either by stopping what he is doing or by continuing his efforts. Thus, the total amount of time $\mathcal{N}_T$ that an agent devotes to an activity $\alpha \in \mathbb{A}$ in $\tau_\lambda$ — being $\lambda$ a temporal unit, such as “seconds” (s) or “hours” (h) — is equivalent to the number of instants that the actor performs an activity. And this is determined by the cardinal of the set of decisions that the actor has taken in a given time horizon $\mathcal{H}$:

$$\mathcal{N}_T(\alpha, \mathcal{H}, \lambda) = \# \left\{ \tau_\lambda \in \mathcal{H} \mid \psi(\alpha, \tau_\lambda) = \max_{\hat{\alpha}} \psi(\hat{\alpha}, \tau_\lambda) \right\}$$

(5)

Defining $\mathbb{A}$ as the set of size $n$ of activities that an agent can perform, $\mathcal{H}_e\mathbb{A}$ as the set of activities of size $k$ that he actually performs in an interval $\mathcal{H}$ and $\mathcal{T}$ as the total aggregate utility of the latter set $\mathcal{H}_e\mathbb{A}$ in an interval $\mathcal{H}$, we formalise both expressions as:

$$\mathbb{A} = \{ \alpha_1, \alpha_2, ..., \alpha_n \}$$

$$\mathcal{H}_e\mathbb{A} = \{ \frac{\lambda}{k} \alpha_1, \frac{\lambda}{k} \alpha_2, ..., \frac{\lambda}{k} \alpha_k \}$$

$$\mathcal{T}(\mathcal{H}) = \sum_{\tau \in \mathcal{H}} \sum_{\hat{\alpha} \in \mathcal{H}_e\mathbb{A}} \psi(\hat{\alpha}, \tau)$$

(6, 7, 8)

We assume that, in order to get closer to $\mathcal{T}$, the agent will take action at the present time with the objective of trying to maximise his utilities. In sum, he will try to achieve $\mathcal{T}_1(\mathcal{H}) < \mathcal{T}_2(\mathcal{H})$ through actions that he can take at this particular instant in time (as we have seen so far). Note, however, that the total utility accruing to two agents will differ according to their time preference. This is defined as an individual’s predilection to enjoy goods in the present (Frederick, Loewenstein, O’Donoghue, 2002; Doyle, 2013). In sum, the higher the time preference, the less “forward-looking” the agent will be; and vice versa, the lower the time preference, the more vision and concern for the long term the agent will show. This means that, given an agent $\rho_H$ with high time preference and $\rho_L$ for an agent with low-time preference, we have that $\text{card}(\mathcal{H}_{\rho_H}) < \text{card}(\mathcal{H}_{\rho_L})$ and, consequently, $\mathcal{T}(\mathcal{H}_{\rho_H}) < \mathcal{T}(\mathcal{H}_{\rho_L})$. In fact, low time preference agents tend to enjoy more ($\delta_{\rho_L}(\alpha, \tau) > \delta_{\rho_H}(\alpha, \tau)$) and suffer less ($\epsilon_{\rho_L}(\alpha, \tau) < \epsilon_{\rho_H}(\alpha, \tau)$) by doing activities that most people find burdensome (e.g. studying or exercising), so they are more likely to do them. And, following the same logic, they attach lower utilities ($\delta_{\rho_L}^*(\alpha, \tau) < \delta_{\rho_H}^*(\alpha, \tau)$) and higher disutilities ($\epsilon_{\rho_L}^*(\alpha, \tau) > \epsilon_{\rho_H}^*(\alpha, \tau)$) to activities whose sole object is instant gratification. Thus, we could say that $\mathcal{N}_T(\alpha^H, \mathcal{H}_{\rho_H}, \lambda) < \mathcal{N}_T(\alpha^L, \mathcal{H}_{\rho_L}, \lambda)$, where $\alpha^\pi$ is the professional activity carried out by the corresponding agent (marked with a super index) and $\lambda$ is any unit of time. This leads to the fact that the marginal utility curve of a person of low time preference

1 The activities that an agent can perform also vary according to time (a teenager will not be able to attend to listen to his favourite band sing at 17:00 if the concert is at 21:00), but that factor is ignored here as it is not relevant for the purposes of the study we are proposing to carry out.
will be above and flatter than the marginal utility curve of a person of high time preference for professional and personal tasks (see Figure 2). And this shifting effect of the marginal utility curves will have an effect on the number of productive hours carried out by an economic agent, since the flatter its $\mu$ the less effect an improvement in the quality of life of individuals will have when it comes to reducing the length of their productive hours. Thus, people with a higher time preference will tend to be much more successful and achieve a higher academic and financial status, as we will see later.

![Marginal Utility of Activity α](image)

**FIGURE 2.** Effective marginal utility over time. The black line symbolises $\psi_{FREE}(\alpha, \tau)$, i.e. the marginal utility of an activity $\alpha$ before the appearance of the tax. The dashed line plots $\psi_{TAX}(\alpha, \tau)$, i.e. the marginal utility of the activity $\alpha$ after the application of the tax burden. The grey area represents the efficiency loss resulting from the additional tax.

### 2.3. Elasticity of the labour market according to changes in income level

Through the use of the model, by performing an analysis of its marginal effective utility curves, we can analyse how much the tax burden —income reduction— or a higher monetary influx —income increase— affects the willingness to work off the country’s labour force, as well as try to predict what the population would do if their standard of living increased. It should be noted that the decision-making we proceed to analyse is a tendency analysis: if the user has incentives to act in a certain way, we conclude that he will try to do so when the opportunity arises. In this article, it is not of scientific interest to us whether he actually succeeds or not.

Note that our aim is to scrutinise the actions of individuals rather than their achievements (to see what the actor does to place himself in a job rather than to consider whether he actually achieves it and to evaluate such achievement or failure). Thus, a worker might be discriminated against or held in low esteem by his or her bosses —making it difficult, for example, for the worker’s demands for shorter working hours to be met— without altering
our analysis: remember that we are interested in what the actor is willing to do and does if the opportunities are favourable. As we shall see below, not only will people with lower purchasing power and lower wages not be able to leave their jobs for clear reasons, but also the new, additional incentive to leave is very little compared to those with high purchasing power or productive capacity, who are much more impelled to work less (as we shall see below). It is these incentives that are the focus of the following discussion, rather than the material and social conditions that may facilitate the attainment of a particular goal, and this premise should be borne in mind when reading the rest of this study.

With respect to taxes, we know that if we create or increase the income tax burden, oriented to the productive activity of an economic agent, the marginal utility that he assigns to each monetary unit varies, given the considerable change in $\mu(\alpha, \tau)$ and $\phi(\alpha, \tau)$. Because of this, as each unit of time in his work (and in all economic activity) will yield him a lower monetary utility, he will be driven to work comparatively fewer hours at the margin. That is, there will be a loss of efficiency equivalent to the area between the utility function in the instant before the tax $\psi(\alpha, \tau)$ and after the tax $\psi_\phi(\alpha, \tau)$. Thus, the efficiency loss $L$ of realising $\alpha$ from utility level $u_1$ to $u_2$, will be equal to the integral of the difference between the pre-tax and post-tax effective marginal utility functions, respectively:

$$L(\alpha, u_1, u_2) = \int_{u_1}^{u_2} (\psi(\alpha, t) - \psi_\phi(\alpha, t)) dt$$

(9)

The mathematical tool used to calculate the efficiency loss given an increase in user income has to be considered differently, since what we observe in $\psi(\alpha, \tau)$ is the same function, but having undergone a contraction, so that in a given interval $\mathcal{I}$, the original function prior to the increase in income $\psi_\Omega(\alpha, \tau)$ and its subsequent $\psi_\Omega^*(\alpha, \tau)$ leads to $\psi_\Omega(\alpha, \tau) \geq \psi_\Omega^*(\alpha, \tau)$ for every $\tau \in \mathcal{H}$. In practice it is to superimpose the same function $\psi(\alpha, \tau)$ in scale, having the former from an earlier income level than the latter. In other words, representing $\psi(\alpha, \tau)$ in $[0, k]$ —which would result in $\psi_\Omega(\alpha, \tau)$— and in $[0, n]$ —which would result in $\psi_\Omega^*(\alpha, \tau)$— with $k < n$. In sum, when dealing with a change in the level of income we have:

$$L^*(\alpha, u_1, u_2) = \int_{u_1}^{u_2} (\psi_\Omega(\alpha, t) - \psi_\Omega^*(\alpha, t)) dt$$

(10)

In case we want to find the total loss of efficiency (i.e. of the subjective preference and utility with which the actor relates to the corresponding activity) which we denote by $L_T$ and $L_T^*$ for the case of income loss and income gain, respectively, we calculate the improper integral from zero to infinity:
\[ \mathcal{L}_T(\alpha) = \lim_{A \to \infty} \mathcal{L}(\alpha, 0, A) = \int_0^\infty (\psi_\Omega(\alpha, t) - \psi_\Phi(\alpha, t)) dt \]

\[ \mathcal{L}_T^*(\alpha) = \lim_{A \to \infty} \mathcal{L}(\alpha, 0, A) = \int_0^\infty (\psi_\Omega(\alpha, t) - \psi_\Omega^*(\alpha, t)) dt \]

(11, 12)

In the same way, other activities on the margin will become more attractive in the eyes of the agent. Having a lower remuneration for carrying out the same activity, the actor will gradually tend to spend less time on productive-professional activities (those that offer monetary remuneration to the individual) and will spend much more time on non-productive (non-monetary) activities.

More specifically, in those activities that the actor subjectively values more. Thus, the variation of time \( \Omega_T \) devoted to an activity \( \alpha_2 \) with respect to another \( \alpha_1 \) at a fixed time horizon \( \mathcal{H} \) and fixed time magnitude \( \lambda \) will be, using the mid-point formula:

\[ \Omega_T(\alpha_1, \alpha_2) = \left\| \frac{\Delta \mathcal{N}_T(\alpha_1, \mathcal{H}, \lambda)}{\mathcal{N}_T(\alpha_2, \mathcal{H}, \lambda)} \right\| = \left\| \frac{\mathcal{N}_{TFREE}(\alpha_1, \mathcal{H}, \lambda) - \mathcal{N}_{TTAX}(\alpha_1, \mathcal{H}, \lambda)}{[\mathcal{N}_{TFREE}(\alpha_1, \mathcal{H}, \lambda) + \mathcal{N}_{TTAX}(\alpha_1, \mathcal{H}, \lambda)]/2} \right\| \]

(13)

Which is the total aggregate change in preferences at the margin when performing both activities represented in time units. For example, if a father earns $2,000 per month and a new tax arises that affects his wage, altering the marginal utility of going to his job \( \alpha_2 \) and caring for his children \( \alpha_1 \) in such a way that he works 10 percent fewer hours (assuming his job allows him to do so) and spends 20 percent more time with his children, then we have \( \Omega_T(\alpha_1, \alpha_2) = \left\| \frac{0.2}{0.1} \right\| = 2 \). In other words, for every hour previously spent at work, she now spends with her children.

3. Theoretical analysis

The model proposed can be used for numerous investigations. Next, we analyse the variation in \( \mathcal{N}_T(\alpha, \mathcal{H}, \lambda) \) for any productive activity \( \alpha \) (labour) and its consequent loss of efficiency due to (1) the different taxes on personal income — and which of them minimises it—; (2) changes in the level of wealth of the country inhabited by economic agents; (3) direct aid received by them; and (4) time preferences. In this section, we use the model proposed in the previous section to make general predictions about agents’ behaviour with respect to the aforementioned points (labour tax, hours worked according to income...).
3.1. Prosperity effects on efficiency

An economic agent with a specific marginal utility function will vary his behaviour according to the remuneration he obtains from his productive activity. The higher the income, the steeper the slope of \( \iota(\alpha_m, \tau) \) so that the marginal utility \( \mu(\alpha_m, \tau) \) and effective utility \( \psi(\alpha_m, \tau) \) of the profession or labour \( \alpha_m \) will be lower. Because of this, the non-monetary disutilities \( \epsilon(\alpha_m, \tau) \) will far outweigh the monetary utilities \( \delta(\alpha_m, \tau) \) and the actor will slowly tend to perform tasks that are more personally rewarding than financially rewarding. However, if he does not find a sufficiently enriching occupation so that \( \psi(\alpha, \tau) > \psi(\alpha_m, \tau) \), then he will continue in his job even though the perceived utility of each additional monetary unit is lower. Simply because they are still higher than those of any other activity at his disposal.

In practice, this leads to the inference that a generalised increase in the standard of living will lead to a reduction in the number of hours worked, symbolising a “zoom” in the marginal utility function. Figure 3 shows the effect of a large wage increase on a low-paid person. He has an identical utility function in both scenarios, but as his income level changes so does the maximum value of the x-axis he reaches. Plotting both graphs along the x-axis on a scale from 0 to 1, we see that lower pay (black line) spans a smaller range of \( \iota(\alpha_m, \tau) \). For the highest pay, it is the other way around.

![Marginal Utility of Productive Activity](image)

**Figure 3.** Effects of an increase in an individual’s income level. The utility function is the same, but the range it covers is larger (e.g., tripling your salary from $0-$1,000 to $0-$3,000). Along the way, there is an increase in the disutilities of productive activity, much as a single percentage rate of a tax would do.

In sum, other things being equal, an increase in the quality of life of economic agents will lead them to work fewer hours. Only in those cases in which these actors do not find a more entertaining, rewarding or stimulating pastime than work, or the total utility of the remuneration received is much greater than that of carrying out other activities, will the
agent tend to remain at work. In certain cases there may even be an increase in the number of hours, usually due to a high consideration of the activity. For example, a doctor without borders might spend more time on philanthropic work after increasing his or her monetary base, as he or she is better able to help more people.

3.2. Effects of aid and compensation on efficiency
Any additional financial benefit received by an agent, whether direct or in kind, will have the same impact as an increase in the standard of living. As the individual’s level of wealth and income increases, the case is identical to the one analysed above, and therefore the conclusions to be drawn must be identical in both cases.

3.3. Efficiency losses: licences, fees, charges and tranche taxes

Most societies set up their systems for taxing income from economic activity in a staggered manner. It also tends to differ between countries. The effect of these brackets is to convert $\psi_{TAX}(\alpha, \tau)$ into a piecewise function, the magnitude of the efficiency loss of which will differ from one case to another. In each slice, the effect of the tax will be (1) to shift the function and (2) to contract the concave curve downwards.

To analyse these cases, we will study the behaviour of an employee or self-employed person according to the brackets to which he or she is exposed. To do this, let us imagine a country A whose progressive taxes on personal income are listed in Table 1. In another country B we have the same tax levels and rates as in country A, although this one includes a tax exemption for the first income brackets. In contrast, we have a country C whose tax rate increases with income level; and a country D which maintains a fixed percentage rate. And country E, which charges a licence to work identically at all income levels. We will now analyse the effects this has on the time spent on productive activities.

<table>
<thead>
<tr>
<th>Income (in 1,000 USD)</th>
<th>Country A</th>
<th>Country B</th>
<th>Country C</th>
<th>Country D</th>
<th>Country E</th>
</tr>
</thead>
<tbody>
<tr>
<td>0-10</td>
<td>10%</td>
<td>0%</td>
<td>30%</td>
<td>15%</td>
<td>1</td>
</tr>
<tr>
<td>10-50</td>
<td>20%</td>
<td>20%</td>
<td>20%</td>
<td>15%</td>
<td>1</td>
</tr>
<tr>
<td>&gt;50</td>
<td>30%</td>
<td>30%</td>
<td>10%</td>
<td>15%</td>
<td>1</td>
</tr>
</tbody>
</table>

TABLE 1. Income tax rate by income level in each country under analysis.

3.3.1. Countries A and B

Given the similarity of the two countries tax systems, we will analyse them side by side. In both cases, the immediate consequence of their configuration is a (1) disincentive to work overtime and, in general, a (2) disincentive to the agent’s pursuit of higher levels of income and profits. This does not necessarily mean that actors will instantly modify their behaviour: if they are strongly constrained by their job agreements and scarce opportunities then
presumably they will stay in their positions, deeming unemployment a worse situation than a lower remuneration. Yet, the tendency will remain, and, if they had the chance, i.e. when disutilities from income taxes are greater than perceived utility from income, then agents would inevitably go off their jobs (although this only happens in extreme and unrealistic scenarios, such as a 90% or higher income tax). This means that the staggered level means that the marginal utility curve is only slightly modified for small and medium-wage workers since the alteration of the curve in these cases will not be as drastic (see Figure 4). In contrast, for subsequent positions requiring greater responsibility and entailing higher pay, there is a significant decrease in the agent’s willingness to work the same hours (in case he/she is in a high-paying job), or to aspire to a new one with those characteristics. That is, the rate of new firm creation, entrepreneurial attitude and productivity of the most productive agents — assuming that their higher income is due to higher productivity or quality of their work (Messenger, Lee, McCann, 2007) — is exponentially reduced, although this does not immediately affect the preferences of agents with lower income.

Similarly, the difference between the graph for countries A and B is noteworthy: almost zero. The reason for this is that, in the first income brackets, the marginal utility will be very high and will remain almost unchanged: the curvature of the function will be very similar, so it will only move downwards. This leads to the fact that, in case the actor does not find jobs to which the premium does not apply (which does not happen, since the tax applies to all jobs available to the agent), then the agent will continue to devote the same amount of hours and effort to his work as before the tax. However, it does shift the curve by a fixed amount, since the remaining income to be received from his economic activity (\((\alpha, \tau)\)) also shifts by the amount specified by the tax.

**Figure 4.** On the left, we have the graph corresponding to the tax rate of country A; on the right, the graph corresponding to country B. They show \(\psi_{FREE}(\alpha, \tau)\) (black line) over \(\psi_{TAX}(\alpha, \tau)\) (dashed line). Significant steps denote the passage from one income level bracket to another.
3.3.2. **Country C**

The agent is exposed to regressive taxation, in which the percentage of his income going to tax payments diminishes the more he earns. In this case, the shape traced by $\psi_{\text{TAX}}(\alpha, \tau)$ is stepwise upwards: he loses efficiency exponentially but at a slower rate the wider his income becomes. In other words, $\psi_{\text{TAX}}(\alpha, \tau)$ approaches $\psi_{\text{FREE}}(\alpha, \tau)$ in “leaps” (Figure 5). The consequences of regressive taxation are perhaps the most interesting for the tax collector. The actor finds himself with a lower initial income but receives higher amounts for each additional monetary unit beyond a certain threshold. In that sense, $i(\alpha, \tau)$ takes a “jump upwards” after each crossing of the tax bracket.

Since the actor does not find any job with which to escape the tax burden (since it applies to all), he will continue to work the same number of hours and with the same intensity as before for the first marginal income units. Not having a better alternative but needing the money, he will remain in his job. However, due to the tax advantages of increasing his income level, he will perceive a marginal utility much higher than earning a little more. And, assuming that higher wages correspond to higher labour skills and higher productivity, the agent will be incentivised to acquire better training and improve the quantity of outputs he produces. Likewise, the effect on the creation of new companies and other activities that potentially raise the level of income to a large extent will be very small, affecting to a lesser extent the robustness and development of the business fabric. Thus, the incentive of a regressive tax is to increase productivity, skills, hours worked and, ultimately, the income level of individuals.

It may also be the system with the highest revenue collection, given (1) the stimulus it provides to economic activity ($L_T(\alpha)$ of the regressive model is the lowest of the four models analysed) by retaining much of the productivity of agents; and (2) it increases the direct revenue of the State since most of its revenue comes from middle and low-income families. A 2019 IRS report divided taxpayers into two equal-sized groups with taxpayers belonging to the lowest 50% of income earners (middle and lower classes) and taxpayers whose annual remuneration constitutes that of the highest 50% of income earners (upper-middle and upper class). It can be seen that almost 97% of labour tax revenues came from the first group (middle and lower classes), and only about 3% of tax revenues from the second group (upper-middle and upper class) (Internal Revenue Service, Rettig, Johnson, 2020).

In that sense, a regressive tax could increase state remuneration by taking advantage of a lower output effect in the economy, as well as the unchanged predisposition of workers to remain in their jobs. While this does not imply that it is politically desirable.
3.3.3. **Country D**

In case of facing a fixed tax rate, the agent will see a (1) shift of his marginal utility curve; as well as a (2) contraction of the marginal utility curve (steeper curve) corresponding to the amount of the tax. In this case (see Figure 6), the efficiency loss acts at all levels in a similar way to the progressive brackets, although it is composed of a single larger bracket. If small, the tax will not have a large effect on the behaviour of actors at different income levels (if anything, perhaps a little more on actors with higher purchasing power). However, as it increases, the efficiency loss is exponentially reduced across the entire graph in the same way as would be experienced in the corresponding bracket in country A. Thus, the difference between the two approaches is that, in country A, the efficiency loss is borne by the higher-income agents; whereas in country D, the efficiency loss is distributed across all agents. However, since a flat rate usually has an intermediate level in the tier bracket (i.e. 15% instead of 30% —top end— or 10% —bottom end—) the total efficiency loss ends up being smaller ($L^A_T(\alpha) > L^D_T(\alpha)$). However, if country D adopts the upper end of country A’s tranches, its efficiency loss will be comparatively larger than that of the country ($L^A_T(\alpha) < L^D_T(\alpha)$). Thus, if the flat tax approach is adopted, it would have to be at a substantially lower bracket than the upper bracket that the country would adopt if it opted for a progressive tax.
FIGURE 6. Change in the marginal utility received by agents in country D. The flat percentage rate causes actors to pay in absolute terms more in taxes the more they earn, leading to a contraction of the curve.

3.3.4. Country E

The effect of a fixed licence or fee on the agent’s preferences is to shift down the curve, but not to contract it (Figure 7). In sum, the returns to productive activities will be comparatively lower than doing other chores. This could lead to a linear loss of relevance of labour in the eyes of economic agents, the effect being smaller than that of a progressive tax bracket (provided that the flat tax is small). However, it might reduce the creation of new entrepreneurial projects (especially small ones, when the actor does not intend or perceive the possibility of accumulating large profits from his entrepreneurial research), since the agent might not have the certainty of systematically earning enough money to cover the fee, as well as having a surplus that constitutes his personal remuneration.
In sum, we can conclude that a tax on labour places a relatively light burden on the number of hours worked by a small and medium-sized economic agent (see Figure 8), given that the utility they receive from each initial monetary unit is comparatively almost as high (workers, even if they earn less, still want to pay their mortgages and feed their offspring). However, a pernicious effect of the tax for extended hours can be seen. The model predicts that taxes cause the number of overtime hours worked by an actor to vary exponentially, as the marginal benefit of each additional unit of time spent on the productive activity is drastically reduced, in turn increasing the marginal utility of non-monetary chores (such as childcare). This may affect the number of new firms that emerge in a country, as well as the number of entrepreneurial projects undertaken by economic agents (e.g. the expansion of a medium-sized firm), these being negative externalities resulting from the new tax, although this does not have such a strong impact on the hours spent by the ordinary worker.

3.4. Temporary preference for long-term performance/success

According to our model, people with a greater ability to defer instant gratification are likely to achieve greater professional and personal success in their lives. Thus, if we consider a person with a low time preference $\rho_L$ who is able to take into account a very long period of time—he is concerned about the education he can give to his children, grandchildren, and even the legacy he can leave—and another $\rho_H$ whose time preference is high and therefore short-sighted—he does not repress his instincts and seeks the most immediate enjoyment: the student who parties instead of studying for an exam, the father who allows his offspring to drink alcohol at a very young age, the teenagers who use tobacco or drugs at the cost of deteriorating their bodies, etc.—, we will see that the decision making of the two agents will be very different.
Assuming that instant gratification assumes that the disutilities of labour $\alpha_{\sigma}$ are lower, then $\varepsilon_{\rho_{L}}(\alpha_{\sigma}, \tau) < \varepsilon_{\rho_{H}}(\alpha_{\sigma}, \tau)$. If we consider that the utility of labour increases and those of unproductive activities $A_{\theta}$ decreases, then we also have $\delta_{\rho_{L}}(\alpha_{\theta}, \tau) > \delta_{\rho_{H}}(\alpha_{\theta}, \tau)$ for every $\alpha_{\theta} \in A_{\theta}$. Both agents are considered to perceive the same marginal utility from each monetary unit so that $\eta_{\rho_{L}}(\alpha_{\sigma}, \tau) = \eta_{\rho_{H}}(\alpha_{\sigma}, \tau)$. Finally, given that agent $\rho_{L}$ is much more resilient than $\rho_{H}$, then we have that the opportunity cost of engaging in productive activity for $\rho_{L}$ will be much higher than for $\rho_{H}$. And the other way around for leisure activities. Hence, we have that the greater the difference between the actors’ time preferences, the magnitude of the above functions will widen to a large extent. Likewise, the more long-term looking agent will have a longer time horizon than the high time preference agent, so that $H_{\rho_{L}} > H_{\rho_{H}}$ for activities that do not provide instant gratification or instant profits. And conversely, $H_{\rho_{L}} < H_{\rho_{H}}$ for activities belonging to $A_{\theta}$. So a person with a really low time preference $\rho_{L}$, and another one whose time preference is really high $\rho_{H}$, then $\ll$ and $\gg$ could be used instead of $<$ and $>$. 

With all these assumptions, we can closely follow the decision-making of each agent. Suppose that both actors have the same routine, and even the same life, and that the only thing that distinguishes them is their time preference. In that case, faced with the same maximisation problem as in equation (4):

$$\max_{\alpha} \psi(\alpha, \tau)$$

(4re)

They will make decisions at time $\tau$ to maximise $T(H)$. However, if we consider $H_{\rho_{L}} > H_{\rho_{H}}$ for productive activities and vice versa for leisure activities, denoted by the subscripts we have used so far, then by sheer size we will have that $T_{\sigma}(H_{\rho_{L}}) > T_{\sigma}(H_{\rho_{H}})$ and $T_{\theta}(H_{\rho_{L}}) < T_{\theta}(H_{\rho_{H}})$. And this leads the actor $\rho_{L}$ to opt more for productive activities than $\rho_{H}$. In sum, $\kappa_{T}(\alpha_{\sigma}, H_{\rho_{L}}, \lambda) > \kappa_{T}(\alpha_{\sigma}, H_{\rho_{H}}, \lambda)$ and $\kappa_{T}(\alpha_{\theta}, H_{\rho_{L}}, \lambda) < \kappa_{T}(\alpha_{\theta}, H_{\rho_{H}}, \lambda)$. Recall its definition according to equation (5):

$$\kappa_{T}(\alpha, H, \lambda) = \# \left\{ \tau_{\lambda} \in H \mid \psi(\alpha, \tau_{\lambda}) = \max_{\alpha} \psi(\alpha, \tau_{\lambda}) \right\}$$

(5re)

Some research has been conducted on this line. However, it is difficult to assign an average cardinal value to countries’ citizenship to determine how long or short their time horizon is. Some authors —such as Eugen von Böhm-Bawerk and Irving Fisher— proposed to consider interest rates as a reflection of individual time preferences (Samuelson, 1994; Hayes, 2021). However, as their successors point out, they cannot act as such when they are exogenously determined by government administrations (Shostak, 2022). Other
Methods have been proposed (Arrondel, Masson, 2007; Hardisty, et al., 2013; Angerer, et al., 2015; Tasoff, Zhang, 2021), but none of them seem to be useful for the analysis of this study.

Special mention should be made of the research work of Banfield (1974), who identifies the close relationship between the social class of each citizen—as he defines them: upper class, middle class, working class and lower class—with their future orientations and preferences. Equally helpful is the extensive literature in the field of psychology pointing to the ability to defer instant gratification as one of the fundamental keys to an individual’s long-term performance and prosperity (e.g., Funder, Block, Block, 1983; Mischel, Shoda, Rodriguez, 1989; Shoda, Mischel, Peake, 1990; Casey, et al., 2011; Haynes, et al., 2022).

4. Conclusions

The model infers that increasing the standard of living of the population causes the number of hours worked by the population to fall or tend to fall, given a diminishing marginal utility of financial income. For someone whose career ambitions and gratification from doing what they do are very high compared to their family and leisure aspirations, earning more money will incentivise them to work more. However, for the majority of the population with a lower obsession with work (either because it does not generate as much non-monetary utility from doing it, or because they find it boring, unimportant, etc.), then they will tend to work less comparatively than they did before, given a compression of the marginal utility curve.

This tendency does not ensure they will end up doing so, since if they do not have the alternative then workers must decide between abandoning their jobs or carrying on in their conditions, and most of them will presumably choose the latest option. In this sense, the increase in living standards has a similar effect and the provision of direct aid, although less pernicious in most cases (given that households’ real income rise and not all actors lose efficiency, and even in certain cases it’s gain is incentivised—for example, with young entrepreneurs, for whom it is easier and quicker to prosper in an environment where the bulk of their potential customers have high purchasing power—), than that of a tax. In addition, there is a noticeable flattening of the marginal utility curve for those with much longer time horizons and less for those who are not as forward-looking and disciplined. Therefore, people and countries will work more proportionately the fewer barriers, impediments and taxes on professional activity are lower. However, the majority will put in a certain base number of hours—those who need the remuneration most—even if taxes are high, as they require a minimum to subsist and, ideally, to prosper.
Similarly, there is greater sensitivity on the part of agents further to the right of the utility function. This means that tax rates will distort agents’ claims and objectives more the higher their wealth level, pointing to a possible recommendation for policymakers: do not over-step personal income taxation. Otherwise, this will discourage foreign investors from settling in the country as well as encourage domestic entrepreneurs to leave, given their greater predilection and ambitious interest in preserving their capital.

The validity of these conclusions remains unchanged even in the recent pandemic context. The pandemic did not fundamentally alter in any way the design of the model we have presented, which presumably applies to all situations and scenarios. However, the quarantine stage of COVID-19 did lead to fundamental changes in the utility functions of the agents, who saw a huge increase in the total costs (monetary and non-monetary) of going to work (among other reasons, because it was very inconvenient or even outright illegal) and also gained greater satisfaction from staying at home (enjoying full attention among family members). These are the only changes that could be expected from the components of the model in an unusual environment, such as that of the massive spread of the aforementioned disease.

Other common scenarios in which the general structure and functioning of the model are maintained are in cases of extreme poverty, abundant informal economy, or even in cultures more prone and idolatrous to work or rest. Where poverty is severe and the informal economy is high due to high taxes, this reflects a (1) very high total cost to foreign investors, who would lose much of their profits via taxes and would not find workers legally given the conditions in the country; and (2) a situation of need on the part of the labour force, who given their poverty cannot avoid working long hours (as is often the case in less developed countries) for the sake of pocketing some remuneration. Also, if income taxes are excessive, citizens may find it more profitable to enlist in companies under unofficial agreements and receive taxes in the black, thus favouring the informal economy as is to be expected in such a situation. Concerning cultural differences, as there may be between the West and the East, depending on how they are configured, this will lead to a generalised perception of non-monetary benefits and costs more or less favourable to work or family, as appropriate. Thus, the more industrious and disciplined peoples are likely to be more productive and capable than those less ambitious, while spending less time establishing and strengthening family relationships in favour of commercial ones. In all these cases, the model describes well the behaviour of all the actors involved, with only the values and expressions of the four functions we have seen has to be varied to reflect reliably the decisions to which the actors are propelled.

**Competing interests**
The author(s) declare none.
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